Overview of GEMS® Equity Model Formulae and Parameters

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Equity Equations

$$\frac{dS(t)}{S(t)} = [(r(t) - D(t)) + \mu_0 + \mu_1 V(t) - \lambda m V(t)]dt + \sqrt{V(t)}dW_1(t) + \gamma dN(t)$$

$$dV(t) = \big(\alpha - \beta V(t)\big)dt + \sigma\sqrt{V(t)}dW_2(t)$$

$$dD(t) = \kappa\big(\alpha_D + \theta z(t) - D(t)\big)dt + \sigma_D\sqrt{D(t)}dW_3(t) + D(t)\gamma_D dN(t)$$

Where;

- r(t) is the short rate simulated by the interest rate model
- D(t) is the stochastic dividend yield
- μ_0 and μ_1 are risk premia
- V(t) is the stochastic variance
- N(t) is a Poisson counting process with intensity $\lambda_l V(t)$ which is shared by the price and dividend processes
- γ is a variable jump size
- m is the average of the variable jump size γ
- β is a parameter controlling the mean reversion speed of the variance process
- σ is the instantaneous variance of the variance process
- κ is a parameter controlling the mean reversion speed of the dividend process
- α_D is a parameter controlling the mean reversion level of the dividend process
- z(t) is a government bond yield scaled by parameter θ
- σ_D is the instantaneous variance of the dividend process
- γ_D is the variable jump size of the dividend process

Equity Equations

Easiest to break it down into our categories:

1. Risk Premium Parameters
2. Variance Parameters
3. Jump Parameters
4. Dividend Parameters

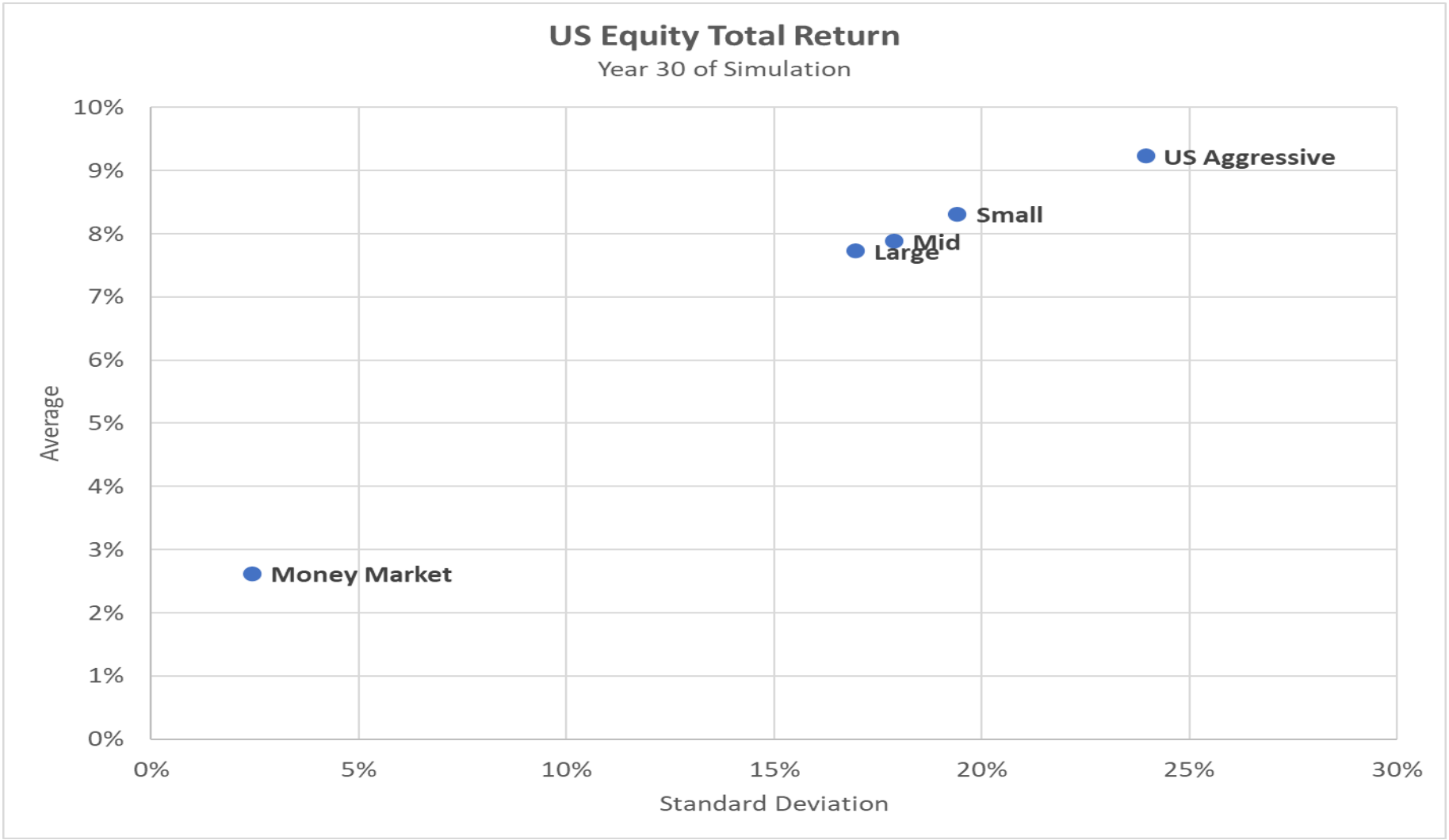
Equity Equations

Easiest to break it down into our categories:

1. Risk Premium Parameters

Risk Premium Parameters	Large Cap	Mid Cap	Small Cap	Aggressive US
Fixed Return ($\mu 0$)	0.05193343	0.054419634	0.058658186	0.065662379
Risk Premium Coefficient ($\mu 1$)	0.092564524	0.001976953	0.001216322	0.018593352

Equity Returns



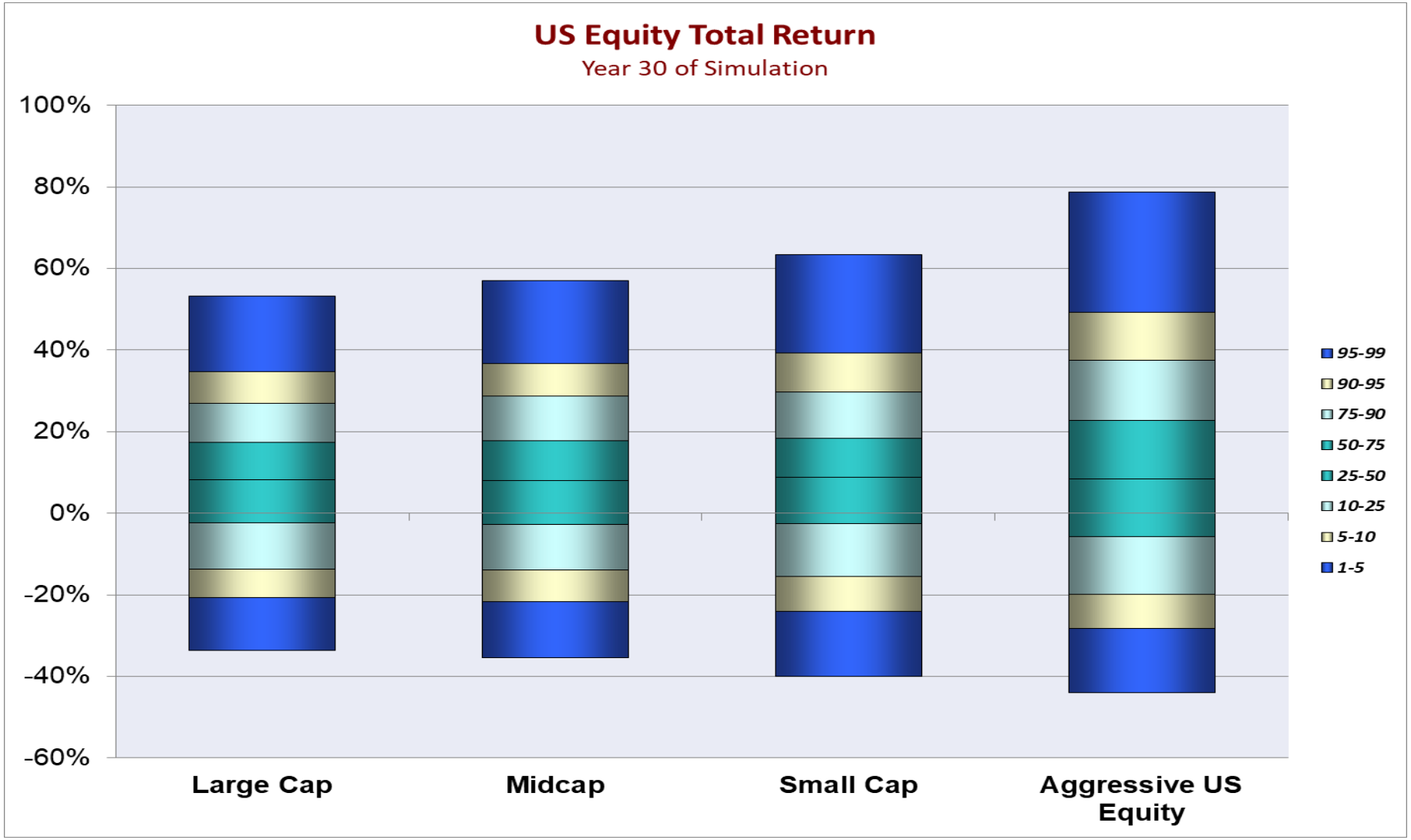
Equity Equations

Easiest to break it down into our categories:

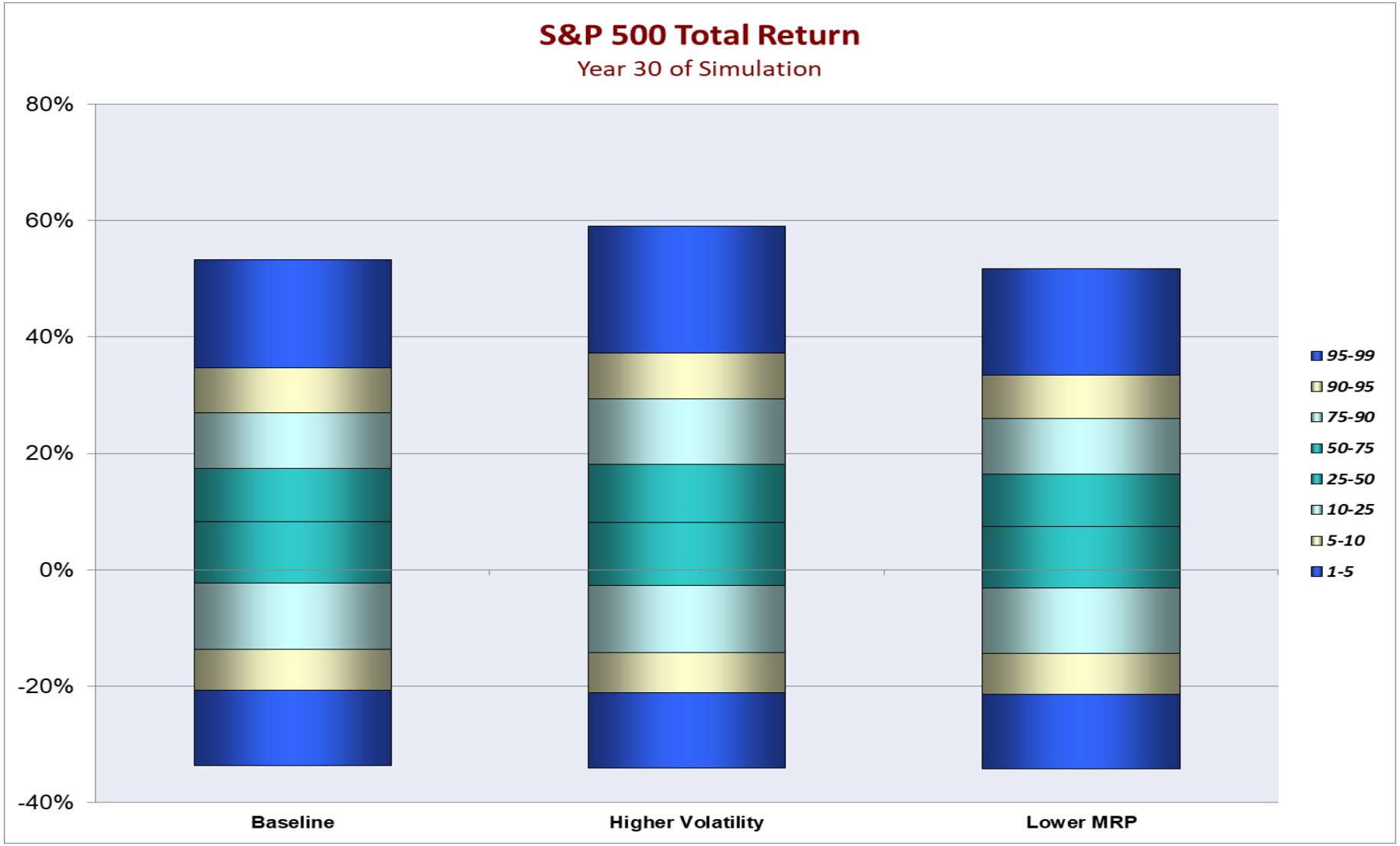
1. Risk Premium Parameters
2. Variance Parameters

Risk Premium Parameters	Large Cap	Mid Cap	Small Cap	Aggressive US
Initial Value	0.010408794	0.014820659	0.015796296	0.018894151
Alpha (α)	0.00556428	0.004701873	0.004907813	0.008586708
Beta (β)	0.396577448	0.261910925	0.278360478	0.307103203
Sigma (σ)	0.081871925	0.077045982	0.096470077	0.090934214

Equity Returns



Equity Returns – Impact of Treasury Calibration



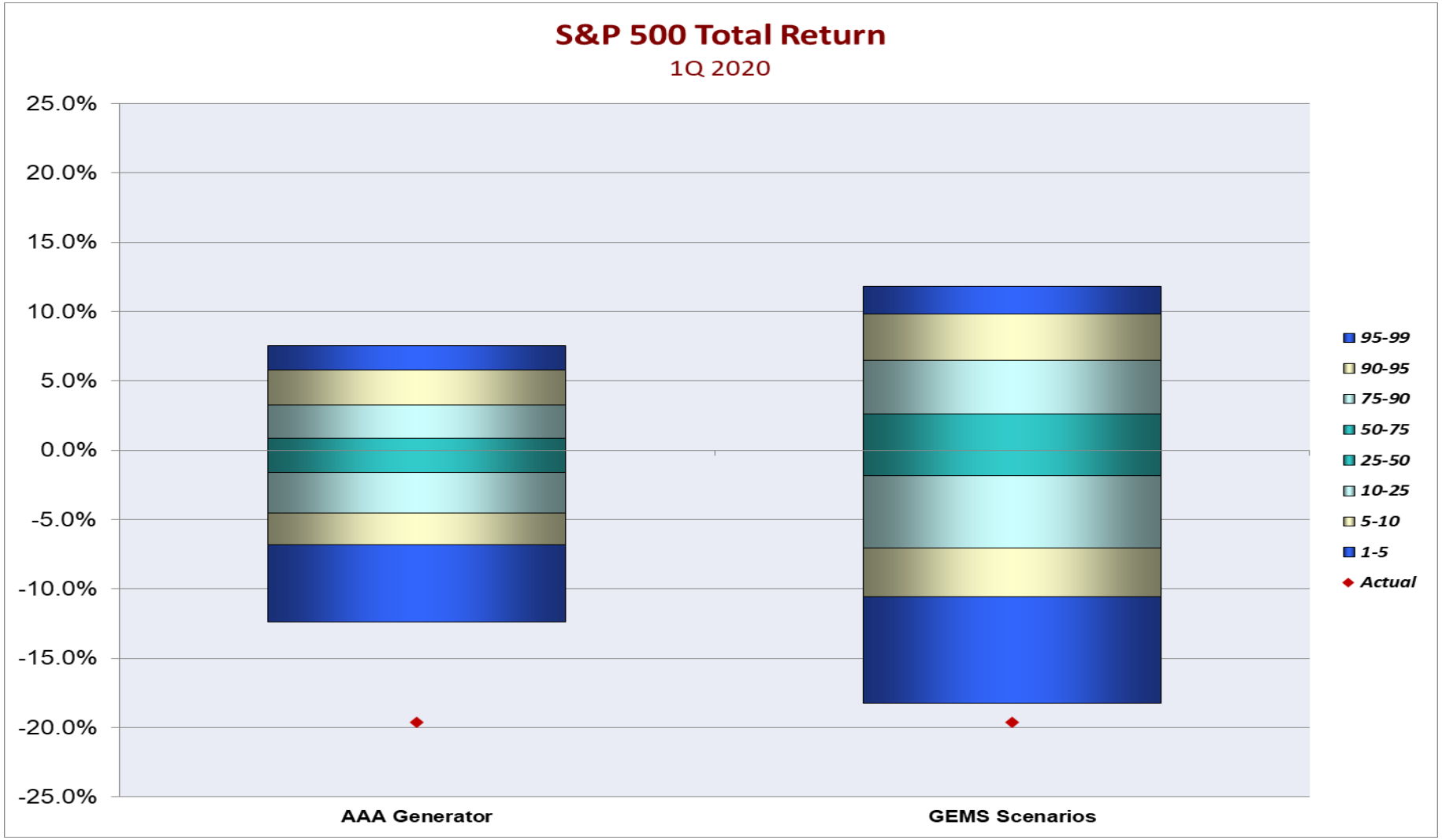
Equity Equations

Easiest to break it down into our categories:

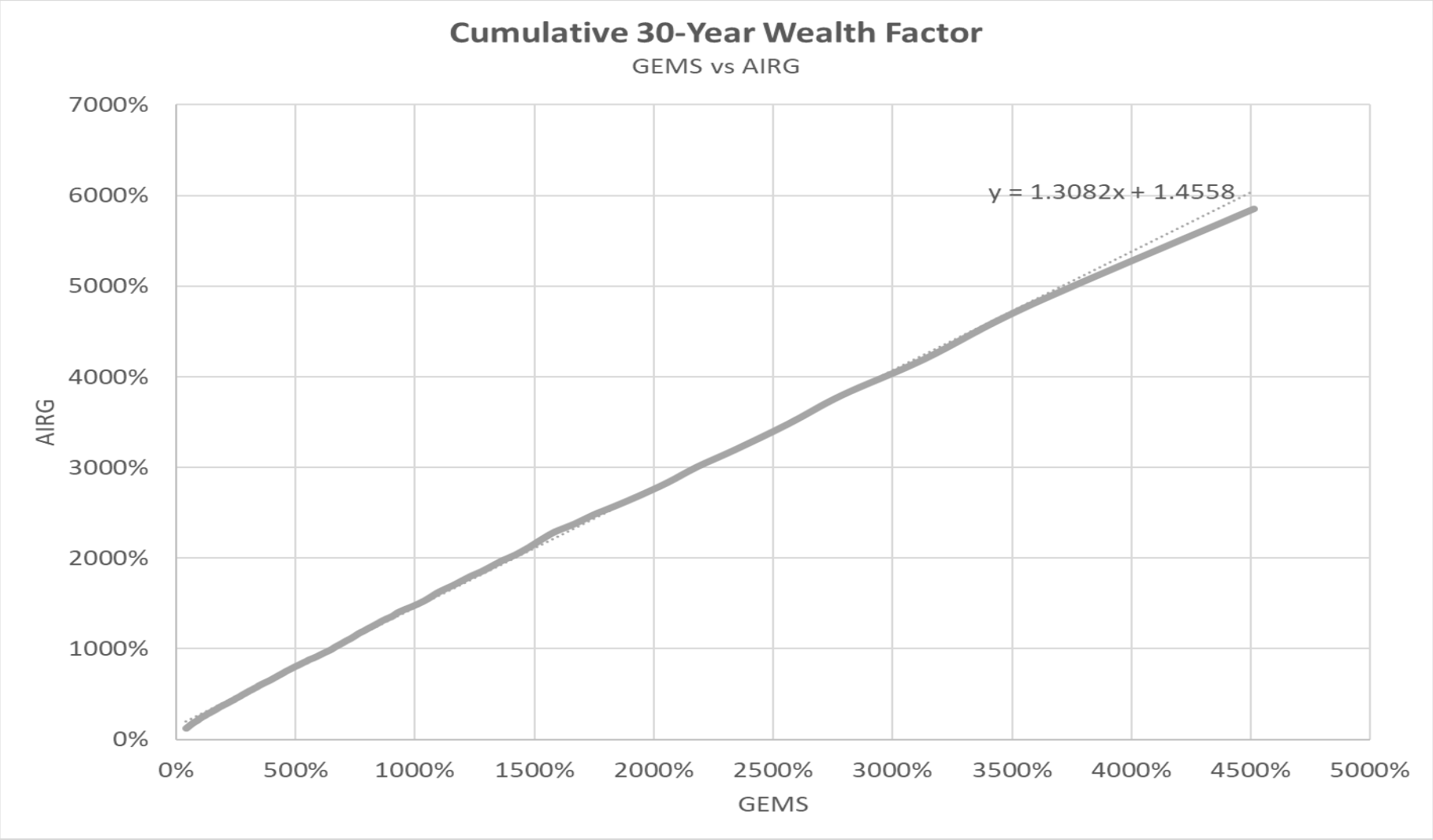
1. Risk Premium Parameters
2. Variance Parameters
3. Jump Parameters

Risk Premium Parameters	Large Cap	Mid Cap	Small Cap	Aggressive US
Variable Jump Intensity (λ1)	139.5881991	113.4167777	112.9783715	128.7243452
Jump Size Mean (m)	-0.052498034	-0.042004029	-0.069631949	-0.050422735
Jump Size Volatility	0.0575	0.05749	0.057488	0.05949

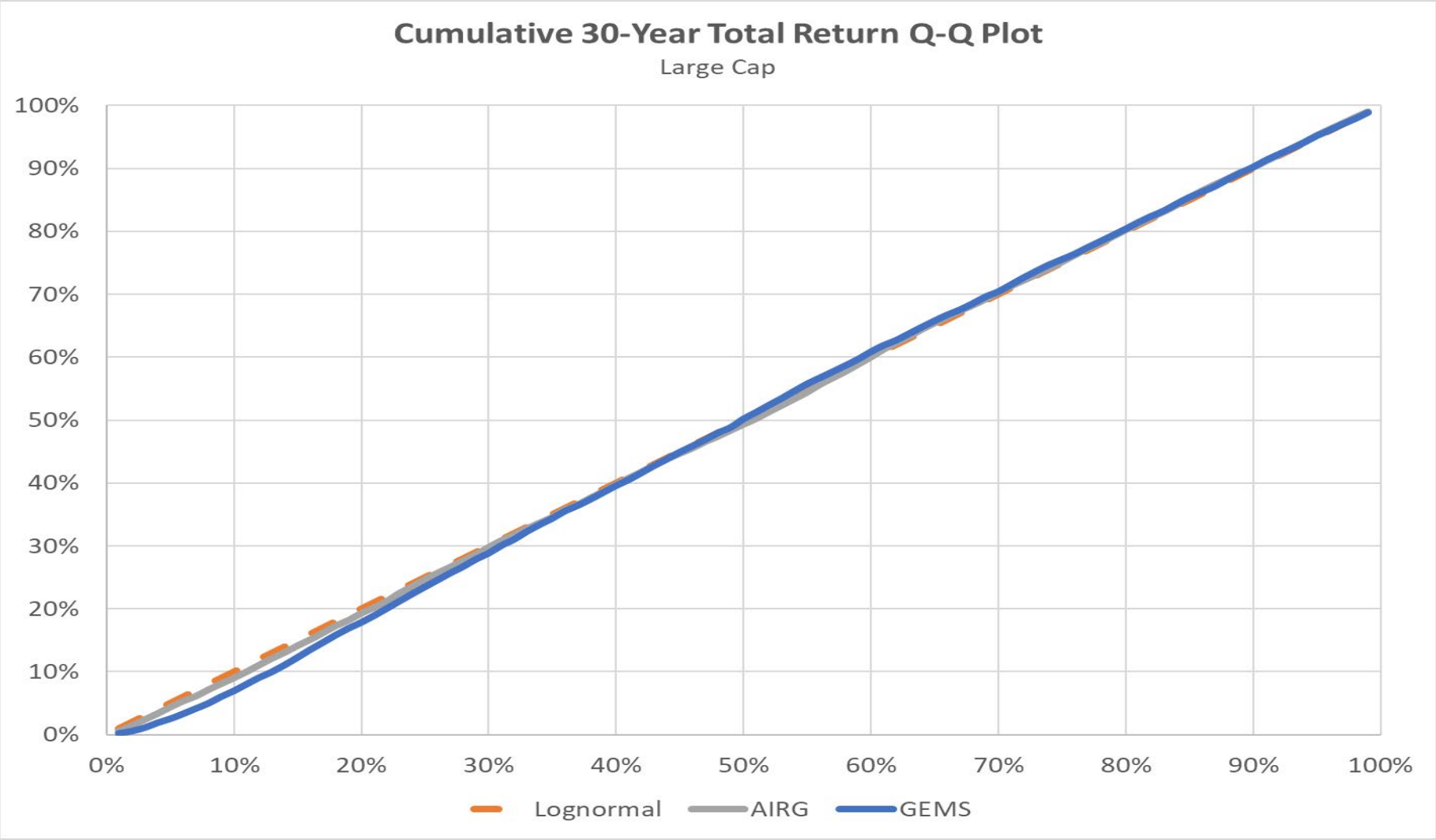
Equity Equation - Impact of Jumps



Equity Equation - Impact of Jumps



Equity Equation - Impact of Jumps



Equity Equations

Easiest to break it down into our categories:

1. Risk Premium Parameters
2. Variance Parameters
3. Jump Parameters
4. Dividend Parameters

Risk Premium Parameters	Large Cap	Mid Cap	Small Cap	Aggressive US
Alpha (α_D)	0.013265383	0.008360257	0.01212561	0.006587937
Kappa (κ_D)	0.461711897	0.330378827	1.149459043	0.550540254
Theta (θ_D)	0.005172487	0.045636438	0.001348106	0.066856412
Sigma (σ_D)	0.030871345	0.020316506	0.047428824	0.019789501

Equity Equations

Correlations:

Corr	LCP	LCD	LCV	MCP	MCD	MCV	SCP	SCD	SCV	OCP	OCD
LCD	-17%										
LCV	-48%	21%									
MCP	96%	-21%	-48%								
MCD	-14%	92%	27%	-22%							
MCV	-55%	34%	89%	-53%	41%						
SCP	96%	-21%	-45%	96%	-23%	-54%					
SCD	-23%	69%	47%	-30%	80%	65%	-33%				
SCV	-50%	31%	85%	-48%	40%	98%	-50%	68%			
OCP	96%	-10%	-48%	90%	-5%	-50%	93%	-12%	-45%		
OCD	-42%	48%	53%	-40%	36%	50%	-33%	21%	37%	-38%	
OCV	-46%	4%	94%	-44%	10%	86%	-41%	32%	82%	-48%	46%